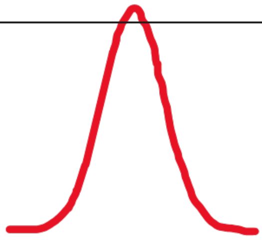
17 - Confidence interval 2

Given population distribution with mean μ, the sampling distribution of size n looks like:

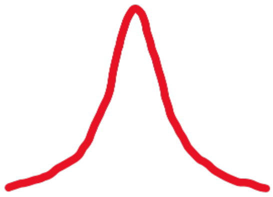
For σ known:



Sampling distribution of $\bar{x}$ is normal with $\mu_{\bar{x}}=\mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Sampling distribution of $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ is normal with μ = 0 and σ = 1.

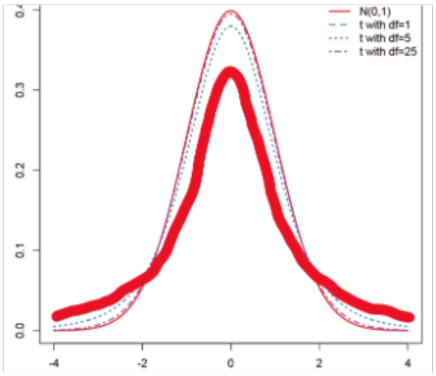
For σ unknown:



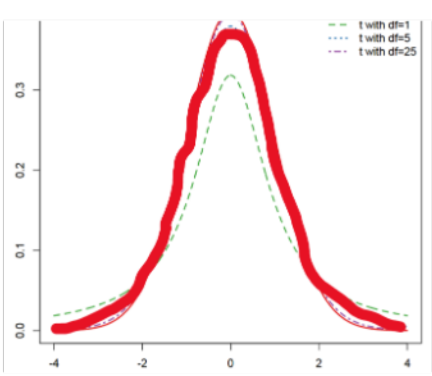
Sampling distribution of $t = \dfrac{\bar{x}-\mu}{s_x/\sqrt{n}}$ is the t_{n-1}-distribution.

The df=n−1 is the degrees of freedom (df) of the t_{n-1}-distribution.

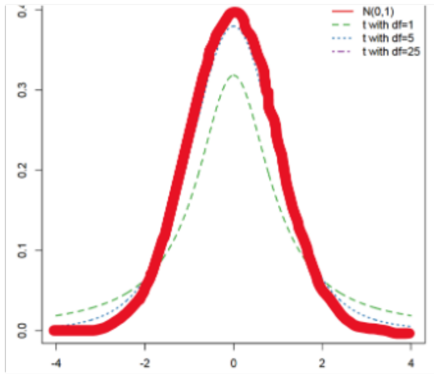
Student's t_{n-1}-distribution is a family of symmetric curves fatter than the normal curve. As sample size n increases, it becomes more normal.



t_1-distribution



t_5-distribution



t_{25}-distribution

Facts about Confidence Interval of Mean with σ unknown

The c-confidence interval for population mean μ obtained from an SRS with sample mean $\bar{x}$ and sample sd s_x is $\bar{x} - E < \mu < \bar{x} + E$

with margin of error $E = t_{c,n-1} \cdot \dfrac{s_x}{\sqrt{n}}$

when the following conditions are met:
10% Condition: $N \geq 10n$.
Approx. normal: Pop. distribution is normal or $n \geq 30$.

Here $t_{c,n-1} = \text{qt}((1\text{-c})/2, \text{df} = \text{n-1})$ (R command)

Example 1. Construct a 95% confidence interval for mean NOX exhaust level in all US cars given a sample of 40 readings with mean 1.268 and sample sd 0.333.

Answer.
$N \geq 10 \cdot 40$ and $n \geq 30$ so we can proceed.

$c = 95\% \Rightarrow t_{c,n-1}$ = **qt(0.025, df = 39) = -2.023**

$$E = 2.023 \frac{0.333}{\sqrt{40}} = 0.107$$

$$1.268 - 0.107 < \mu < 1.268 + 0.107$$

Conclusion: We are 95% confident that the true mean NOX exhaust level is between 1.161 and 1.375.